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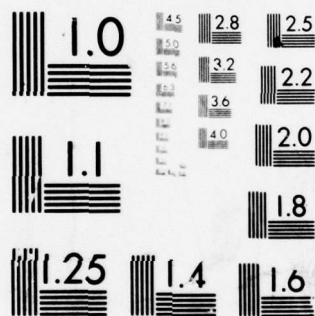
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by

Ernest H. Forman
Nozer D. Singpurwalla

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26 June 1978

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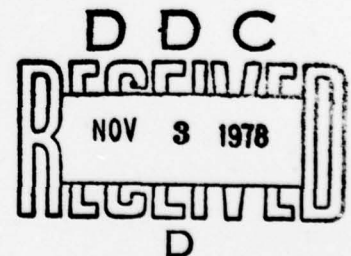
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1. Introduction and Summary

Research into improving and measuring computer software reliability has progressed along several different directions. Typical of these are structured programming, proofs of correctness of programs, and the stochastic analysis of software failure data [cf. Amster and Shooman (1975)].

In this paper we shall focus attention on certain decision-theoretic aspects of the software reliability problem. These aspects arise quite naturally when we consider a statistical analysis of software failure data. In particular, we shall develop a procedure for testing the hypothesis that a given software system contains no errors, and in the sequel, determine an optimal interval of time for which the software has to be exercised in order to test this hypothesis. The overall organization of our paper is as follows.

In Section 2, we shall briefly review a simple probabilistic model for describing software failures. This model is due to Jelinski and Moranda (1972) and has also been described by Lloyd and Lipow (1977) p. 516. In Section 3 we shall present some results pertaining to the estimation of the parameters of the model discussed in Section 2. We shall also present in Section 3 an empirical stopping rule which signals the end of the debugging phase for a given software system. This stopping rule has been recently proposed

by us in the open literature [see Forman and Singpurwalla (1977)]. In Section 4 we shall discuss a test of the hypothesis that the software contains no more errors, and determine an optimal interval of time for which the program has to be exercised in order to test the hypothesis.

The material in Sections 2 and 3 can be regarded as expository, whereas the material in Section 4 is new and represents the *raison d'être* of this paper.

2. The Model by Jelinski and Moranda

Jelinski and Moranda (1972) have proposed a model for describing failures of computer software. Variations of this model have been considered by Shooman and others [Shooman et al. 1972a; Shooman 1972b, 1973] in several contexts. The applicability of this model for analyzing software failure data from the Apollo program and from a certain system of the U.S. Navy have been discussed by Jelinski and Moranda. Other applications of this model have been described by Moranda (1975).

Let us denote the initial error content in a large software system, such as an operating system, by N ; N is, of course, unknown. By assumption, the failure rate at any point in time is proportional to the residual number of errors in the software. Thus, if τ_1, τ_2, \dots , denote the time points at which software errors are detected and corrected, then the failure rate at any time point between τ_{i-1} and τ_i is $(N-i+1)\phi$, where ϕ is some unknown constant of proportionality.

In Figure 2.1 we show the behavior of the failure rate for this "de-eutrophication" process.

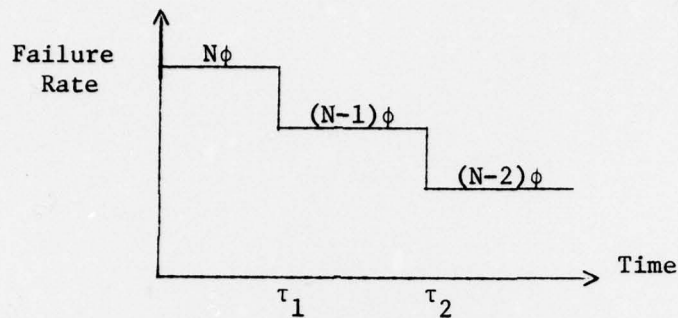


Figure 2.1. Time Behavior of the Failure Rate

Let $T_i = \tau_i - \tau_{i-1}$, $i = 1, 2, \dots$, with $\tau_0 \equiv 0$; then the distribution function of the times between failures T_i is $F(t_i) = P(T_i \leq t_i)$, where

$$F(t_i) = 1 - \exp(-(N-i+1)\phi), \quad \phi > 0, \quad t_i > 0.$$

Let t_1, t_2, \dots, t_n be the realizations of T_1, T_2, \dots, T_n , respectively. Given t_1, t_2, \dots, t_n , a natural objective is to estimate ϕ and N . Note that n is always less than or equal to N . A more pragmatic objective is to obtain a "stopping rule" for debugging the software. The stopping rule should be such that we can be reasonably well assured that the software contains no errors. As will be pointed out in the next section, the above two objectives will be concurrently achieved.

3. Parameter Estimation and an Empirical Stopping Rule

As is discussed in detail by Forman and Singpurwalla (1977), the estimation of N and ϕ , and the development of a stopping rule are based upon an analysis of the behavior of the likelihood function $L(\phi, N)$, where

$$L(\phi, N) = \prod_{i=1}^n (N-i+1)\phi \exp(-(N-i+1)\phi) .$$

Let $L(\phi, N)$ denote the natural logarithm of $L(\phi, N)$, $T = \sum_{i=1}^n t_i$
and $k = \sum_{i=1}^n (i-1)t_i$.

Since N takes only integer values, the unique maximum likelihood estimator of N , say \hat{N} , is that value of N which simultaneously satisfies

$$n \log \left[\frac{(N-1)T-k}{TN-k} \right] + \log \left(\frac{N}{N-n} \right) \geq 0$$

(3.1)

and

$$n \log \left[\frac{(N+1)T-k}{TN-k} \right] + \log \left(\frac{N-n+1}{N+1} \right) \leq 0 .$$

Given \hat{N} , the maximum likelihood estimator of ϕ , is

$$\hat{\phi} = \frac{n}{\hat{N}TN-k} . \quad (3.2)$$

3.1 Properties of the Maximum Likelihood Estimators. The estimators given by Equations (3.1) and (3.2) are quite straightforward to obtain, and have been discussed by Jelinski and Moranda (1972) and by Shooman (1973). However, the fact that when n is much smaller than N , \hat{N} is *highly misleading* has been grossly overlooked. Specifically, when the quantity k/T is small, \hat{N} tends to be unrealistically large, and furthermore, a slight decrease in k/T leads to a disproportionately large increase in \hat{N} . Thus, for small values of k/T , the maximum likelihood estimator of N , \hat{N} is *very unstable*, and may lead to *erroneous conclusions*.

As k/T becomes large, that is, if the times between failures during the latter stages of testing are greater than those during the earlier stages, \hat{N} tends to be close to n , the observed number of failures. Thus $\hat{N} \approx n$ is an indicator of the fact that the program is close to being debugged, and

should therefore provide us with a stopping rule. However, as has been pointed out by Forman and Singpurwalla (1977), it is possible that $\hat{N} \sim n$ and yet the true value N may be far from \hat{N} . Thus $\hat{N} \sim n$ is not always conclusive of the fact that the program is close to being debugged and, therefore, has to be interpreted with caution. For a more conclusive analysis we will have to examine the behavior of the likelihood function in greater detail. This is discussed in the next section.

3.2 The Relative Likelihood Functions and a Stopping Rule. The *relative likelihood function* of N , $R(N)$ is defined as

$$R(N) = \frac{L(N, \hat{\phi}(N))}{L(\hat{N}, \hat{\phi})}$$

where

$$\hat{\phi}(N) = \frac{n}{TN-k}.$$

Note that the shape of $R(N)$ is a function of n , k , and T . We will need to compare $R(N)$ with the *normal relative likelihood function* of N , $R_{\text{normal}}(N)$ defined as

$$R_{\text{normal}}(N) = \exp \left[-\frac{1}{2} \frac{(\hat{N}-N)^2}{\text{Var}(\hat{N})} \right]$$

where

$$\text{Var}(\hat{N}) = \frac{n}{n \sum_{i=1}^n \left(\frac{1}{\hat{N}-i+1} \right)^2 - \left(\sum_{i=1}^n \frac{1}{\hat{N}-i+1} \right)^2}.$$

In the light of our previous discussions, plus some Monte Carlo analyses performed by Forman (1974), we shall propose the following steps which constitute a stopping rule.

1. Compute \hat{N} , the maximum likelihood estimator of N using Equation (3.1).
2. If $\hat{N} \approx n$, proceed to Step 3; if $\hat{N} \gg n$, observe another failure interval t_{n+1} , and go back to Step 1 above.
3. Compute $R(N)$ and $R_{\text{normal}}(N)$ for various values of N , and see if the plots of $R(N)$ and $R_{\text{normal}}(N)$ are in good agreement with each other. If the two plots show a large disparity, then \hat{N} is a misleading estimator of N . When this happens, observe another failure interval t_{n+1} and repeat the above steps. If the plots of $R(N)$ and $R_{\text{normal}}(N)$ show good agreement, then $\hat{N} \approx n$ is a good estimator of N and we do not have to test the software further to obtain t_{n+1} .

An example illustrating the application of the above stopping rule to some real data on software failures is given in Forman and Singpurwalla (1977).

4. A Test of the Hypothesis that the Software Contains No Errors and an Optimal Time Interval for Testing

Let us assume that a given software has been subjected to the debugging process and that all the steps of our proposed stopping rule have been satisfactorily undertaken. A potential user of this program will be interested in answers to the following two questions.

1. What is the assurance that the program contains no more errors?
2. How much additional testing should be done in order to achieve a specified assurance?

Clearly, an answer to the above questions will be a function of the risks that the user is willing to take, the cost of testing, and the consequences of software failure during its operation.

In what follows, we shall attempt to answer the above questions in a quantitative manner. We shall attempt to answer the first question by formulating it as a problem of testing hypothesis.

4.1 Testing of the Hypothesis. Let N^* denote the number of errors which are remaining in a debugged program. Let our null hypothesis be H_0 , where

$$H_0 : N^* = 0 ,$$

versus the alternative hypothesis H_1 , where

$$H_1 : N^* = r , \quad r = 1, 2, \dots ,$$

In order to test the above hypotheses, we shall exercise the software for an additional t_a units of time, and reject H_0 if a failure is encountered. Note that for such a test, the probability of rejecting the null hypothesis when it is true, is zero, since, when $N^* = 0$, we will not encounter any failures. Thus, for our test the so-called *Type I error* is zero.

Let β denote the power of our test; that is, β is the probability of rejecting the null hypothesis when it is false. Since

$$\beta = P[T \leq t_a | N = r] = 1 - \exp(-\phi r t_a) , \quad (4.1)$$

the power of the test can be made as large as is desired by increasing t_a .

If the user is willing to specify a β , then we can calculate t_a by choosing $r = 1$. When this is done, our test procedure will have a power of at least β .

For convenience, we shall denote the dependence of t_a on β by $t_a(\beta)$.

To summarize, our test of hypothesis will proceed along the following lines.

The user will specify a β , the power of the test, and given β , we shall determine $t_a(\beta)$ using Equation (A.1). We shall test the software for $t_a(\beta)$ period of time, and accept H_0 if no failures are encountered during that period; otherwise, we shall reject it.

4.2 Choosing t_a Based on Cost Considerations. We can also choose the t_a discussed above based on cost considerations and the mission time t_m .

Let C_1 be the cost per unit time for testing the software; suppose that C_1 is a constant. Let C_2 be the cost incurred by the failure of the software during the mission time t_m ; C_2 is also assumed to be a constant. Later on, we shall assume that C_1 changes with time. Three outcomes are possible:

- i. The software fails during the additional testing time t_a , in which case the total cost is

$$C_1 t \quad 0 \leq t \leq t_a$$

where t is the time at which failure occurs;

- ii. The software does not fail during the additional testing time, but fails during its operation at some time t ; when this happens, the total cost is

$$C_1 t_a + C_2 \quad t_a \leq t \leq t_a + t_m;$$

- iii. No failure of the software is encountered; the total cost is

$$C_1 t_a \quad t_a + t_m < t.$$

The total expected cost is therefore

$$E(C) = \int_0^{t_a} C_1 t \phi e^{-\phi t} dt + \int_{t_a}^{t_a + t_m} (C_1 t_a + C_2) \phi e^{-\phi t} dt \\ + \int_{t_a + t_m}^{\infty} C_1 t_a \phi e^{-\phi t} dt .$$

In order to solve for t_a , we shall minimize $E(C)$. Since

$$\frac{d}{dt_a} E(C) = e^{-\phi t_a} \left[C_1 - \phi C_2 \left(1 - e^{-\phi t_m} \right) \right]$$

we claim that when $C_1 > \phi C_2 \left(1 - e^{-\phi t_m} \right)$, the value of t_a at which $E(C)$ is minimized is zero. That is, when the cost of testing is much larger than the cost of an operational failure, no additional testing is necessary. However, a potential user may still wish to test the program for $t_a(\beta)$ units of time and be assured that the power of his test is at least β .

If, on the contrary, $C_1 < \phi C_2 \left(1 - e^{-\phi t_m} \right)$, then $t_a = \infty$ minimizes

$E(C)$. This means that we should test exercise the software for an indefinite amount of time. Here, again, a potential user may test for $t_a(\beta)$ units of time and get an assurance that the power of his test is at least β .

The assumption that C_1 is a constant may not be realistic in many situations. In addition to this, this assumption may lead us to the case of indefinite testing as is shown above. We shall now relax this assumption and explore the consequences.

Suppose that the cost of testing is $C_1(t)$, where $C_1(t)$ is a convex non-decreasing function of t . Let us denote the derivative of $C_1(t)$ evaluated at $t = 0$ by $C_1'(0)$.

We can now verify that when

$$C_1'(0) \geq \phi C_2 \left(1 - e^{-\phi t_m} \right)$$

$t_a = 0$ will minimize $E(C)$. If this happens, we can choose $t_a(\beta)$ as our additional test time.

If on the contrary

$$C_1'(0) < \phi C_2 \left(1 - e^{-\phi t_m} \right)$$

then $t_a = C_1'^{-1} \left[\phi C_2 \left(1 - e^{-\phi t_m} \right) \right]$ will minimize $E(C)$; $C_1'^{-1}(\cdot)$ is the inverse of $C_1'(\cdot)$.

As an example, if $C_1(t) = C_1 e^{\alpha t}$, for some $\alpha > 0$, then,

$$t_a = \frac{1}{\alpha} \log \left[\frac{C_2 \phi \left(1 - e^{-\phi t_m} \right)}{C_1 \alpha} \right].$$

For the above situation, if $t_a \geq t_a(\beta)$, and should we test for t_a units of time, then we will not only be minimizing the total expected cost, but will also achieve a power of at least β . If $t_a < t_a(\beta)$, then $C_1 \left(e^{\alpha t_a(\beta)} - e^{\alpha t_a} \right)$ represents the increase in the cost of testing to achieve a power of at least β .

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